

# Deep inelastic scattering at low x: Generalized vector dominance and the color dipole picture\*

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We summarize recent work on low-x deep inelastic scattering. The generalized vector dominance/color-dipole picture (GVD/CDP) implies a scaling behavior for  $\sigma_{\gamma^* p}(W^2, Q^2) \cong \sigma_{\gamma^* p}(\eta)$ , with  $\eta = (Q^2 + m_0^2)/\Lambda^2(W^2)$  and yields an excellent representation of the experimental results on  $\sigma_{\gamma^* p}(\eta)$ .

Two important observations [1] were made on deep inelastic scattering (DIS) at low values of the Bjorken scaling variable  $x_{bj} \cong Q^2/W^2 \ll 1$ , since HERA started running in 1993:

i) The diffractive production of high-mass states (of masses  $M_X \lesssim 30\text{GeV}$ ) at an appreciable rate relative to the total virtual-photon proton cross section,  $\sigma_{\gamma^* p}(W^2, Q^2)$ . The sphericity and thrust analysis [1] of the diffractively produced states revealed (approximate) agreement in shape with the final state found in  $e^+e^-$  annihilation at  $\sqrt{s} = M_X$ . This observation of high-mass diffractive production confirms the conceptual basis of generalized vector dominance (GVD) [2] that extends the role of the low-lying vector mesons in photoproduction [3] to DIS at arbitrary  $Q^2$ , provided  $x_{bj} \ll 1$ .

ii) An increase of  $\sigma_{\gamma^* p}(W^2, Q^2)$  with increasing energy considerably stronger [4] than the smooth “soft-pomeron” behavior known from photoproduction and hadron-hadron scattering. This latter observation may have appeared to be unexpected from the point of view of GVD. A careful analysis, taking into account the quark-antiquark ( $q\bar{q}$ ) configuration in the  $\gamma^*(q\bar{q})$  transition, as well as two-gluon exchange as generic structure of the  $(q\bar{q})p$  interaction [5–7], (compare fig.1), however, reveals [8] that a stronger rise of  $\sigma_{\gamma^* p}(W^2, Q^2)$  with energy than observed in photoproduction is

entirely natural. In fact, one might have predicted a stronger rise than in photoproduction for  $Q^2 \geq m_\rho^2$  in the generalized vector dominance/color dipole picture (GVD/CDP) [6–8] prior to the experimental discovery.

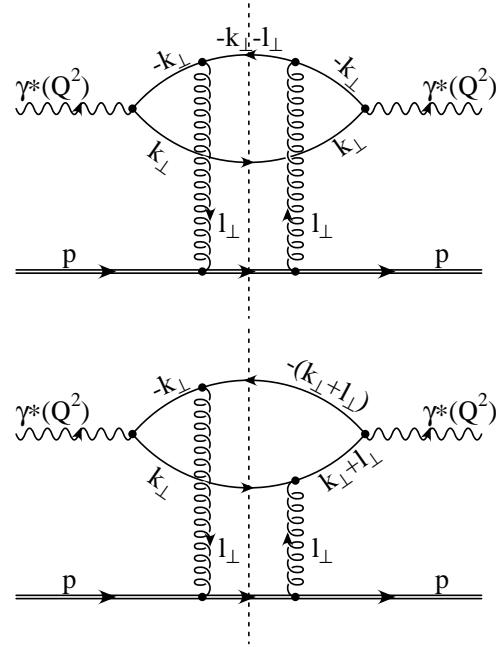


Figure 1. The two-gluon exchange.

We follow recent custom [9–11] in employing the transverse-position-space representation as starting point, rather than proceeding in historic order [6] from momentum space to transverse-

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position space [7]. The representation

$$\begin{aligned} \sigma_{\gamma^* p}(W^2, Q^2) = & \\ \int d^2 r_\perp \int dz |\psi(r_\perp \sqrt{Q^2}, z, Q^2)|^2 & \\ \times \sigma_{(q\bar{q})p}(\vec{r}_\perp^2, z, W^2) & \end{aligned} \quad (1)$$

must be read [7] in conjunction with

$$\begin{aligned} \sigma_{(q\bar{q})p}(\vec{r}_\perp^2, z, W^2) & \\ = \int d^2 l_\perp \tilde{\sigma}(\vec{l}_\perp^2, z, W^2) (1 - e^{-i\vec{l}_\perp \cdot \vec{r}_\perp}) & \end{aligned} \quad (2)$$

Inserting the “color-dipole cross section”  $\sigma_{(q\bar{q})p}(\vec{r}_\perp^2, z, W^2)$  from (2) into (1), together with the Fourier representation of the “photon-wavefunction”,  $\psi(r_\perp \sqrt{Q^2}, z, Q^2)$ , one indeed obtains [7] the generic two-gluon exchange structure: the resulting representation of  $\sigma_{\gamma^* p}(W^2, Q^2)$  is characterized by a linear combination of a diagonal and an off-diagonal term with respect to the masses of the ingoing and outgoing  $q\bar{q}$  states that contribute with equal weight, but opposite in sign, to the virtual Compton forward-scattering amplitude.

The form (2) of the color-dipole cross section (taking the limit of  $r_\perp \rightarrow \infty$ ) implies that the distribution in (the gluon-momentum-transfer variable)  $\vec{l}_\perp$  should tend to zero sufficiently rapidly to yield a convergent result for the integral over  $\tilde{\sigma}(\vec{l}_\perp^2, z, W^2)$ . One may think of introducing a Gaussian in  $\vec{l}_\perp^2$  for  $\tilde{\sigma}(\vec{l}_\perp^2, z, W^2)$ , but a  $\delta$ -function situated at a finite value of  $\vec{l}_\perp^2$  turns out to be equally successful as an effective description of the  $\vec{l}_\perp^2$  dependence of  $\tilde{\sigma}(\vec{l}_\perp^2, z, W^2)$ , and, moreover, its consequences can be fully worked out analytically.

The choice of [8]

$$\begin{aligned} \tilde{\sigma}_{(q\bar{q})p}(\vec{l}_\perp^2, z, W^2) & \\ = \sigma^{(\infty)}(W^2) \delta(\vec{l}_\perp^2 - z(1-z)\Lambda^2(W^2)) & \end{aligned} \quad (3)$$

when converted to  $\vec{r}_\perp$  space according to (2), explicitly realizes

i) color transparency, i.e.

$$\begin{aligned} \sigma_{(q\bar{q})p}(\vec{r}_\perp^2, z, W^2) \rightarrow \Lambda^2(W^2) z(1-z) \vec{r}_\perp^2, & \\ \text{for } z(1-z) \vec{r}_\perp^2 \rightarrow 0, & \end{aligned} \quad (4)$$

as well as

ii) unitarity, i.e.

$$\begin{aligned} \sigma_{(q\bar{q})p}(\vec{r}_\perp^2, z, W^2) \rightarrow \sigma_{(q\bar{q})p}^{(\infty)}(W^2), & \\ \text{for } r_\perp \rightarrow \infty, & \end{aligned} \quad (5)$$

where  $\sigma^{(\infty)}(W^2)$  is to have a weak “hadron-like” energy dependence.

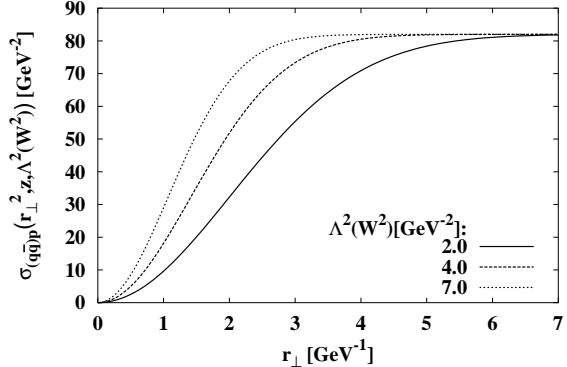


Figure 2. Representation of the  $(q\bar{q})p$  - color dipole cross section for realistic values of  $\Lambda^2(W^2)$ .

Moreover, since the mass difference in off-diagonal transitions,  $M_{q\bar{q}} \rightarrow M'_{q\bar{q}} \neq M_{q\bar{q}}$ , is determined by the magnitude of  $\vec{l}_\perp^2$ , one expects that

iii)  $\Lambda^2(W^2)$  increases with the center-of-mass energy,  $W$ , rather than being a constant, independent of the energy of the  $(q\bar{q})p$  interaction.

The above properties i) to iii) of  $\sigma_{(q\bar{q})p}(\vec{r}_\perp^2, z, W^2)$  are schematically depicted in fig.2, where for simplicity a Gaussian instead of a  $\delta$ -function was assumed, and the weakly  $W$ -dependent function  $\sigma_{(q\bar{q})p}^{(\infty)}(W^2)$ , was replaced by a constant. Figure 2 clearly displays the strong  $W$  dependence, as  $\Lambda^2(W^2)$ , for  $r_\perp \rightarrow 0$ , and the weak one for  $r_\perp \rightarrow \infty$ .

Taking into account

iv) the dependence of  $|\psi|^2$  in (1) on  $\vec{r}_\perp^2 Q^2$ ,

we immediately conclude that with increasing  $Q^2$ , decreasing interquark separations become more dominant. As a consequence, the energy dependence of  $\sigma_{\gamma^* p}(W^2, Q^2)$  becomes increasingly stronger with increasing  $Q^2$ , in agreement with the experimental data [4].

In other words, the GVD/CDP which explicitly incorporates the configuration of the  $\gamma^*(q\bar{q})$  transition, as well as the generic two-gluon-exchange structure for the  $(q\bar{q})p$  interaction, implies the striking change of the  $W$  dependence with increasing  $Q^2$  observed experimentally.

Finally, dimensional analysis, in conjunction with an explicit evaluation of (1) in momentum space, reveals that, in good approximation,

v) the dependence of  $\sigma_{\gamma^* p}(W^2, Q^2)$  is determined by the low-x scaling variable [8]

$$\eta = \frac{Q^2 + m_0^2}{\Lambda^2(W^2)}, \quad (6)$$

where  $m_0$  is a threshold mass  $m_0 < m_\rho$ .

Explicitly,

$$\begin{aligned} \sigma_{\gamma^* p}(W^2, Q^2) &= \sigma_{\gamma p}(W^2) \frac{I(\eta, \frac{m_0^2}{\Lambda^2(W^2)})}{I\left(\frac{m_0^2}{\Lambda^2(W^2)}, \frac{m_0^2}{\Lambda^2(W^2)}\right)} \\ &\cong \sigma_{\gamma^* p}(\eta), \end{aligned} \quad (7)$$

where  $\sigma_{(q\bar{q})p}^{(\infty)}(W^2)$  was substituted in terms of the photoproduction cross section,  $\sigma_{\gamma p}(W^2)$ . For details on (7) we refer to [8]. We only note the representation of the dominant transverse part of the function  $I(\eta, \frac{m_0^2}{\Lambda^2(W^2)})$ ,

$$\begin{aligned} I_T^{(1)}\left(\eta, \frac{m_0^2}{\Lambda^2(W^2)}\right) &= \frac{1}{\pi} \int_{m_0^2}^{\infty} dM^2 \int_{(M-\Lambda(W^2))^2}^{(M+\Lambda(W^2))^2} dM'^2 \\ &\times \left\{ \frac{M^2 \pi \delta(M^2 - M'^2)}{(Q^2 + M^2)(Q^2 + M'^2)} \right. \\ &\left. - \frac{(M'^2 - M^2 - \Lambda^2(W^2))\omega(M^2, M'^2, \Lambda^2(W^2))}{2(Q^2 + M^2)(Q^2 + M'^2)} \right\}, \end{aligned} \quad (8)$$

that displays the underlying structure of GVD and can be shown to approximately coincide with the ansatz of “off-diagonal” GVD [12] from

the pre-QCD era. The explicit expression for  $I(\eta, \frac{m_0^2}{\Lambda^2(W^2)})$  in (7) is complicated, but simple results are obtained in the limits of small  $\eta$ , and of large  $\eta$ , respectively,

$$I \cong I(\eta) = \begin{cases} \ln(1/\eta), & \text{for } \eta \ll 1, \\ \frac{1}{2\eta}, & \text{for } \eta \gg 1. \end{cases} \quad (9)$$

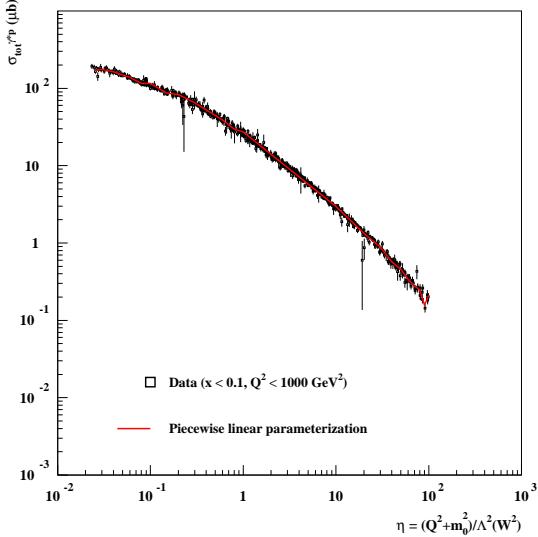


Figure 3. The experimental data for  $\sigma_{\gamma^* p}(W^2, Q^2)$  for  $x \simeq Q^2/W^2 < 0.1$  vs. the low-x scaling variable  $\eta = (Q^2 + m_0^2)/\Lambda^2(W^2)$  (from ref.[8]).

An analysis [8] of the experimental data [4] reveals

i) scaling in  $\eta$  in a model-independent analysis (compare fig.3) that determines  $m_0^2$  and the functional behavior of  $\Lambda^2(W^2)$  in terms of three parameters,  $\Lambda^2(W^2) = C_1(W^2 + W_0^2)^{C_2}$ , and

ii) good agreement of the  $Q^2$  dependence, when  $\Lambda^2(W^2)$  from the model-independent analysis is employed when evaluating  $\sigma_{\gamma^* p}(W^2, Q^2)$  in (7). We refer to [8] for details and only show the results in figs. 4 and 5.

In summary, a unique picture, the QCD-based generalized vector dominance/color-dipole picture (GVD/CDP) emerges for DIS at low  $x_{bj}$

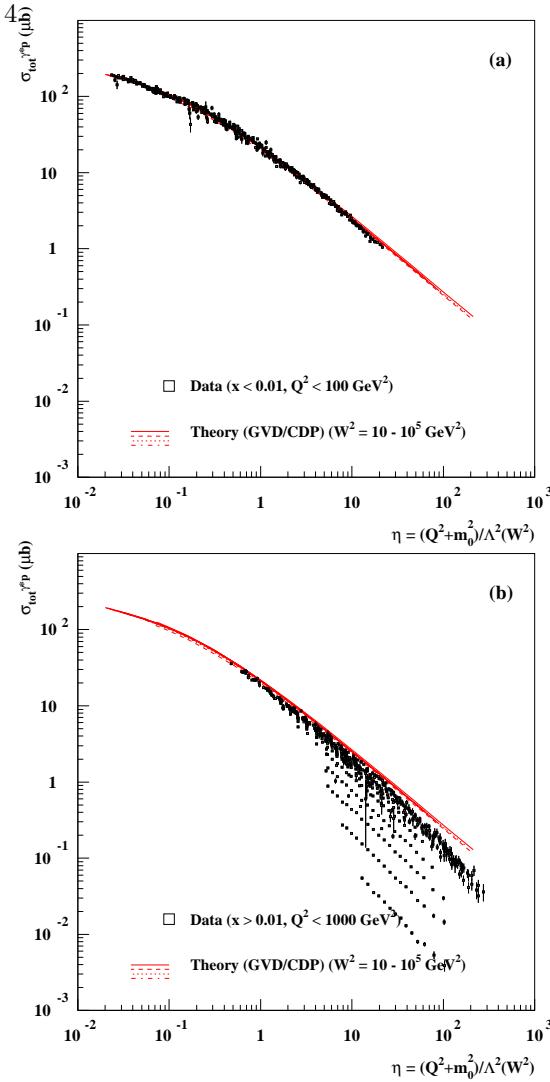


Figure 4. The GVD/CDP scaling curve for  $\sigma_{\gamma^* p}$  compared with the experimental data a) for  $x < 0.01$ , b) for  $x > 0.01$  (from ref.[8]).

and any  $Q^2$ . The incoming virtual photon dissociates into a  $q\bar{q}$ -color-dipole state that propagates and undergoes diffractive forward scattering via an interaction of the generic structure of two-gluon exchange. The DIS experiments at low  $x$  measure the energy dependence of the  $(q\bar{q})$ -color-dipole-proton interaction. With increasing  $Q^2$ , or, equivalently, decreasing transverse interquark separation, the generic two-gluon exchange structure of the  $(q\bar{q})$ -color-dipole-proton

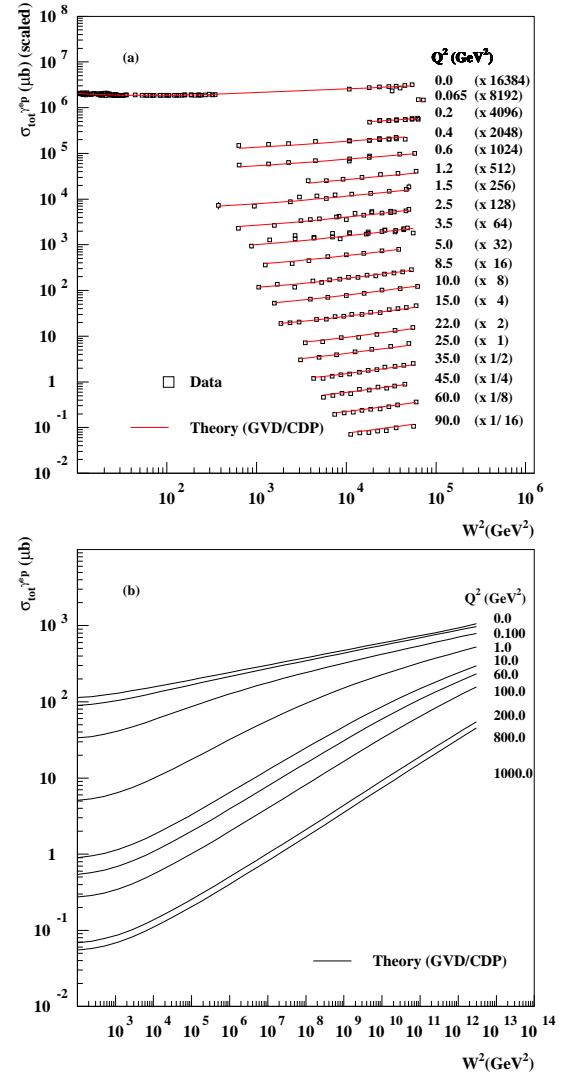


Figure 5. The GVD/CDP predictions for  $\sigma_{\gamma^* p}(W^2, Q^2)$  vs.  $W^2$  at fixed  $Q^2$  a) in the presently accessible energy range compared with experimental data for  $x \leq 0.01$ , b) for asymptotic energies (from ref.[8]).

interaction implies the increasingly stronger  $W$  dependence observed experimentally when  $Q^2$  becomes large. At any  $Q^2$ , at sufficiently high energy, however, the  $W$  dependence will settle down to the unitarity-preserving hadronic one.

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